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A comparison of two stand table projection methods for young *Eucalyptus nitens* (Maiden) plantations in Chile

Guillermo Trincado V.[a,*], Raúl Quezada P.[a], Klaus von Gadow[b]

[a]Institute of Forest Management, Austral University, Casilla 567, Valdivia, Chile
[b]Institute of Forest Management, Georg-August-University Göttingen, Büsgenweg 5, 37077 Göttingen, Germany

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Abstract

The study compares two methods of stand table projection based on data from young *Eucalyptus nitens* (Maiden) plantations in Chile. The projected diameter distributions were estimated using the methods proposed by Nepal and Somers (1992) [For. Sci. 38 (1992) 120] and Cao and Baldwin (1999) [For. Sci. 45 (1999) 506]. The evaluation compared the observed and estimated diameter distributions for different projection intervals, using the Kolmogorov–Smirnov test and an error index called 'relative discrepancy'. The evaluation showed that both methods are suitable for application in the Eucalypt plantations. However, the method proposed by Nepal and Somers proved to be more accurate, especially when the projection period extends over 4 years or more. Expected error and bias for the observed and estimated total and merchantable volumes at stand level were also evaluated. The observed error and bias were relatively low for both methods; however, some differences were detected when the volume distributions were analyzed at diameter class level.
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Keywords: Size class model; Stand table projection; *Eucalyptus*

1. Introduction

Production of roundwood in Chile is based mainly on fast-growing exotic plantations. The most important timber resource in relation to its surface area is radiata pine with plantations covering 1.47 million ha, followed by eucalyptus plantations covering 358.038 ha during the year 2000 (Instituto Forestal, 2001). Currently, there is considerable commercial interest in growing eucalypts. This has led to the increase of plantations areas from 130.915 ha in 1991 to 358.038 ha in 2000 (Instituto Forestal, 1991, 2001). Based on studies about the future availability of wood, the current total growth rate of all planted *Eucalyptus* species in Chile is expected to rise from 2.703 thousand m^3 per year between 1999 and 2000 to 13.702 thousand m^3 per year during the period 2015–2016 (Instituto Forestal, 2000).

Eucalypts are used as raw material for pulp and paper production and therefore the silvicultural management usually does not include thinnings or prunings. Silvicultural research in Chile has focused on determining the most suitable establishment techniques and genetic improvement of the species (Barros et al., 1994). The potential growth rates are high with

* Corresponding author. Current address: Department of Forestry, Virginia Polytechnic and State University, Blacksburg, Virginia 24061, USA.
E-mail address: gtrincad@vt.edu (G. Trincado V.).

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Eucalyptus globulus (Labill) and *Eucalyptus nitens* (Maiden) reaching between 25 and 30 m^3/ha per year (Tibbits et al., 1997).

Considering the high commercial value of the eucalypt plantations, effective planning is essential. Tools for projecting the development of the forest resource are required which includes updating of inventory data to reduce the cost of forest sampling. Of particular importance are tools to project diameter distributions and estimate merchantable timber volumes, thus providing a quantitative basis for economic evaluation of different management options.

Different methods have been used for updating forest inventory data, based on a variety of growth and yield models (see for example Borders and Patterson, 1990; Shortt and Burkhart, 1996; Knowe et al., 1997). Woollons and Hayward (1985), Pienaar and Harrison (1988), Pienaar (1989), Nepal and Somers (1992) and Cao and Baldwin (1999) specifically proposed algorithms to be used in stand table projections.[1] These methods have in common that the diameter data are grouped in diameter classes and that a projection is made for each class. The use of a probability density function is often recommended, the most common being the Weibull distribution function. Knowe et al. (1997) have shown that a suitable stand table projection method, besides producing a future stand table consistent with stand level and individual tree level functions, permits reproduction of multimodal distributions.

The objective of this study is to compare two different methods of stand table projection, i.e. the methods proposed by Nepal and Somers (1992)—denoted here as the NS method—and Cao and Baldwin (1999) denoted here as the CB method, and evaluate their suitability for use in *E. nitens* (Maiden) plantations in Chile.

2. Data and methods

The data used in this analysis were obtained from permanent sample plots located in the geographical region, known as region VIII in central Chile. The plots were established starting in 1985 in *E. nitens* (Maiden) plantations by the Company *Forestal Mininco S.A.* for the research project "Simulation of yield and growth for *Eucalyptus* plantations" A total of 142 plot measurements was available from the 52 plots, totaling 90 projection periods. Between 2 and 7 measurements were available in the different plots. The distribution of re-measurements is presented in Table 1.

The area of the square plots varies between 500 and 1000 m^2. Only plot measurements with a minimum age of 5 years were considered in this study.

Both the CB and NS projection methods are based on the "parameter recovery approach"[2] which is used to estimate the coefficients of the Weibull function (Eq. (1)):

$$f(d_i) = [e^{-((d_i-a)/b)^C}]\left[\left(\frac{c}{b}\right)\left(\frac{d_i-a}{b}\right)^{C-1}\right] \quad (1)$$

where d_i is the midpoint of ith diameter class, a the Weibull location parameter, b the Weibull scale parameter, and c the Weibull form parameter.

Various methods have been used to recover parameters from the three-parameter Weibull distribution (Bailey and Dell, 1973; Burk and Newberry, 1984; Borders and Patterson, 1990; Knowe et al., 1997). The parameter recovery method involves a forecast of stand attributes (such as average diameter, total basal area) and a recovery of the parameters of a theoretical diameter distribution model that will give rise to the stand attributes (Shortt and Burkhart, 1996), thus providing consistency between the derived distribution and the stand values. In this study the method used for the recovery of parameters was similar to the one applied by Nepal and Somers (1992), which was developed by Cao et al. (1982). The parameters of the Weibull distribution are recovered using the average diameter at breast height ($\bar{D}$), the quadratic mean diameter at breast height (D_q) and the minimum diameter at breast height (D_{min}). The parameter a is determined using the following formula:

$$a = D_{min} - \tfrac{1}{2}D_W \quad (2)$$

When the values of the parameter a and the average diameter at breast height ($\bar{D}$) are known, it is possible

[1] The proposed models correspond to the denominated size class models (Vanclay, 1994; Gadow and Hui, 1999).

[2] In the parameter recovery approach, the rates of change of the distribution moments or percentile values at time t are derived from known moments or percentile values and other stand variables at time t(Clutter et al., 1983; Gadow and Hui, 1999).

Table 1
Distribution of measurements in 52 plots by age and stocking

Plot age (years)	Plot basal area (m^2/ha)									
	<20	20–25	25–30	30–35	35–40	40–45	45–50	50–55	55–60	Total
5–6	15	38	34	8						95
7–8		3	8	11	3					25
9–10		1	1	1	3			1		7
11–12			1	1	2	1	1	1	1	8
13–14			1	1		2		1	1	6
>14						1				1
Total	15	42	45	22	8	4	1	3	2	142

to use an iterative procedure to estimate the value of the parameter b from:

$$(\bar{D} - a) = b \times \Gamma\left(1 + \frac{1}{c}\right) \quad (3)$$

Finally the parameter c is obtained by solving the following equation:

$$D_q^2 = a \times \left(a + 2b \times \Gamma\left(1 + \frac{1}{c}\right)\right) + \left(b^2 \times \Gamma\left(1 + \frac{2}{c}\right)\right) \quad (4)$$

where D_W is the width of the diameter class and $\Gamma(\cdot)$ the gamma function.

In this study the data observed at the end of each projection period were used to fit each stand table, thus isolating the projection algorithm effect from the estimated diameter distribution (Borders and Patterson, 1990).

All the stand table projection algorithms were implemented computationally using the SAS (statistical analysis system) and Microsoft Fortran Powerstation v 1.0.

2.1. The Nepal and Somers (NS) algorithm

Nepal and Somers (1992) presented a new algorithm to project stand tables based on the Weibull function (Fig. 1). The method requires a diameter growth equation derived from the Weibull distribution (Bailey, 1980), a current stand table, future basal area, and future survival. The diameter growth equation is used first to project the observed stand table to some future age. Then the resulting stand table is adjusted by an algorithm to ensure that the basal area and survival are matched with the predicted or observed values. The algorithm combines allocation of mortality and any necessary growth adjustments to ensure equality with the stand level estimates.

According to Nepal and Somers (1992) this method is a very good alternative for projecting stands with a multimodal distribution, because an irregular stand structure will be preserved.

2.2. The Cao and Baldwin algorithm

Cao and Baldwin (1999) developed a new method applying a least squares procedure for stand table projection (Fig. 1). The method involves three steps: (a) estimation of survival rates and location of mortality; (b) estimation of the growth for each diameter class; (c) adjustment of the projected diameters through least squares procedures to equal future values of mean diameter and stand basal area. This algorithm can also be used, similar to the NS algorithm, in cases where the diameter structure does not necessarily follow the Weibull distribution.

In their study the CB algorithm produced stand tables that better approximated observed future stand tables than did the NS algorithm. Among the reasons given by the authors for this better performance are the fact that the CB algorithm: (a) applies first a mortality function and then estimates the Weibull parameters considering only surviving trees during the projection period, (b) uses constraints in the adjustment step for three stand attributes (number of trees, basal area and average diameter), in contrast with the NS method which applied constraints only for trees and basal area and (c) allowed the minimum diameter to increase during the projection period.

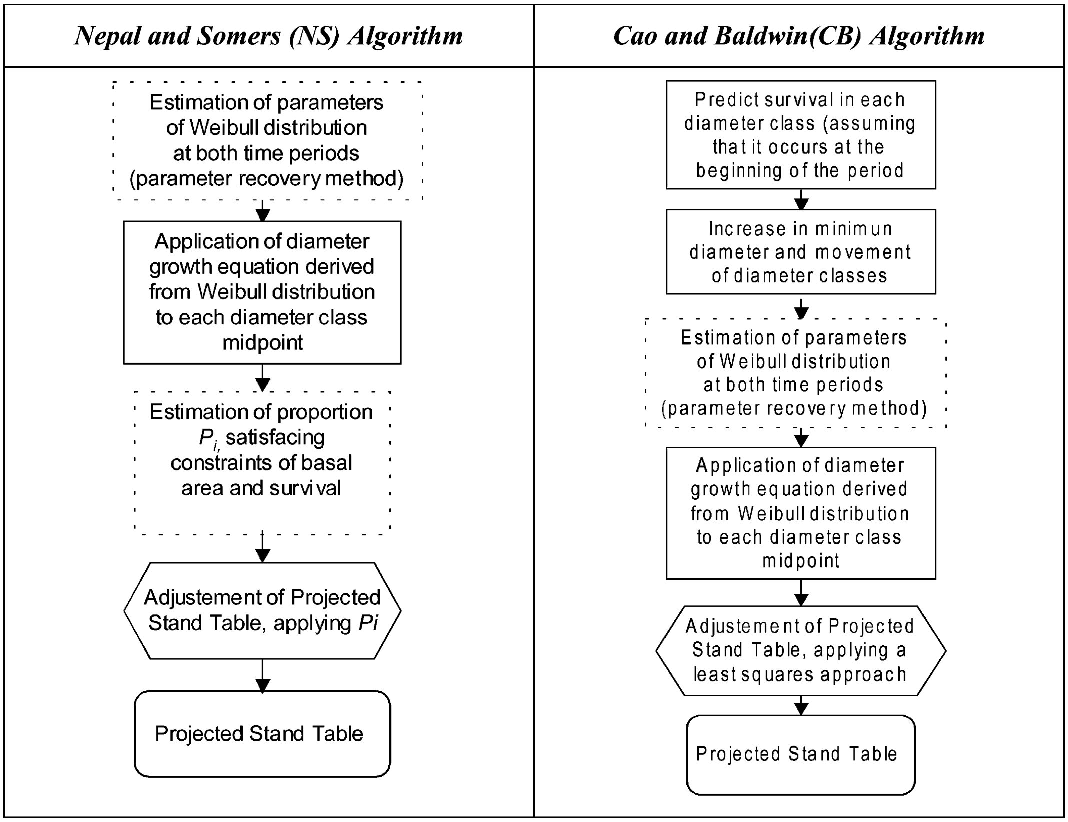

Fig. 1. Flowcharts showing the algorithms proposed by Nepal and Somers (1992) and Cao and Baldwin (1999) for projecting diameter distribution in even-aged forest stands.

3. Model evaluations

The algorithms were compared determining how often observed and estimated distributions do not present a significant difference for the total number of projections (90), using the Kolmogorov–Smirnov (KS) test and measures of maximal cumulative difference (MAD) and an error index (relative discrepancy) applied by Staupendahl (1999), which is similar to the index proposed by Reynolds et al. (1988). The same analysis was carried out for different projection periods (1, 2, 3 and greater than 4 years) to evaluate its effect on the estimation accuracy. Finally comparisons between observed and estimated total and merchantable volume (to a top diameter of 10 cm) based on measures of bias and error were carried out, also analyzing the volume distribution at diameter classes level.

3.1. Diameter distributions

The estimated and observed diameter distributions are compared using the KS test and an error index. The KS test is used to determine if the estimated and observed cumulative distributions are statistically similar and whether the maximum difference is statistically significant (Sokal and Rohlf, 1979):

$$D_{\max} = \max_{-\infty<x<+\infty} |S_{N1}(x) - S_{N2}(x)| \quad (5)$$

where $D_{\max}$ is the maximum difference between the cumulative distributions, $S_{N1}(x)$ the estimated cumulative distribution and $S_{N2}(x)$ the observed cumulative distribution.

The following D value is calculated with n being the number of observations:

$$D = \frac{D_{\max}}{n} \quad (6)$$

The null hypothesis (H_0) states that observed and estimated stand tables are similar without presenting significant differences. The alternative hypothesis (H_1) implies that the observed and the estimated diameter distributions are different. The null hypothesis is rejected when the calculated D value is greater than the tabulated value (D_α).

The D_α, value for large samples can also be calculated using the following formula:

$$D_\alpha = \sqrt{\frac{-\ln((1/2)\alpha)}{2n}} \tag{7}$$

For KS test an acceptance level of 99% ($\alpha = 0.01$) was used.

3.2. Error index (relative discrepancy)

We also use the following error index (known as the '*relative discrepancy*') which measures the relative proportion which would have to be exchanged between the diameter classes of two distributions to obtain the true distribution from the estimated one (Gregorius, 1974; Staupendahl and Puumalainen, 1999):

$$\mathrm{rD} = \frac{1}{2}\sum_{i=1}^{n}|H_i - \hat{H}_i| \tag{8}$$

where rD is the error index or '*relative discrepancy*', $\hat{H}_i$ the estimated relative frequency of diameter class I, H_i the observed or empirical relative frequency of diameter class I and n the number of diameter classes.

Accordingly, $1 - \mathrm{rD}$ is the proportion common to both distributions. A value of 1 means the distributions are completely different, whereas they are identical, if $\mathrm{rD} = 0$. The error index is similar to the one proposed by Reynolds et al. (1988) and used by Borders and Patterson (1990), Nepal and Somers (1992) and Cao and Baldwin (1999) for quantifying the similarity between two diameter distributions. Small rD values indicate a high similarity between the observed and the estimated diameter distributions.

3.3. Total and merchantable volume

This study also considered the capacity of each of the two algorithms to estimate merchantable volumes up to a thin end diameter of 10 cm, obtained from the projected stand tables. The following model was used to determine the mean height of each estimated and observed diameter class at the end of the projection period (Fundación Chile and Instituto Forestal, 2001):

$$\ln(H_i) = b_0 + b_1 H_{\mathrm{d}}^{0.36} + b_2 D_{\mathrm{q}}^{0.36} + b_3 D_i^{-0.7} + b_4 H_{\mathrm{d}}^2 D_i^{-0.7} + b_5 D_{\mathrm{q}} D_i^{-0.7} \tag{9}$$

where H_i is the height (m) for the diameter class I, H_{d} the dominant height (m), D_{q} the quadratic mean diameter (cm) and D_i the diameter (cm) of the diameter class i.

The dominant height was estimated with the following dominant height–age relation:

$$H_{\mathrm{d}} = b_0 \left[1 - \left(1 - \left(\frac{S}{b_0}\right)^{b_1}\right)^{A/20}\right]^{1/b_1} \tag{10}$$

where S is the site index (m) and A the age (year).

The merchantable volume for each diameter class was estimated using the total volume and ratio-volume functions proposed by Honer (1965) and Burkhart (1977), respectively. Each of these functions was previously fitted for the species (Trincado, 1999):

$$\mathrm{TV} = b_0 + b_1 D_i^2 H_i \tag{11}$$

$$V_{\mathrm{d}} = \mathrm{TV}(1 + b_1 d^{b_2} D_i^{b_3}) \tag{12}$$

where TV is the total volume (m^3) inside bark, d the merchantable or top diameter (cm) inside bark and V_{d} the merchantable volume (m^3) inside bark.

The estimated merchantable volumes were compared for the two projection algorithms, using a bias measure (mean residual) and a precision measure (mean absolute residual) as statistical criteria, according to methodology proposed by Borders and Patterson (1990).

4. Results and discussion

Projected diameter distributions were compared for both the total number of observations and the different projection periods. Thus it was possible to determine various effects on the estimations for each method. The effect of each projection method on merchantable volume estimation was also evaluated.

4.1. Stand table projection

The comparison between the two methods was made in order to determine how often the observed and estimated distributions were not significantly different, based on the KS test (Table 2).

Based on KS test, the NS method shows a superior performance in projecting diameter distributions when compared with the CB method.

Table 3 shows the means and standard deviations of the statistics used in the evaluation. The NS method shows less variability and smaller values for the maximum cumulative difference (MAD) between the observed and estimated distribution and for the index error.

This result again indicates a better performance of the method of Nepal and Somers (1992) when compared with the method of Cao and Baldwin (1999).

An important aspect, that has to be considered in the projection of a stand table, is the effect of the length of the projection period on the estimation accuracy. Fig. 2 shows the percentage of KS acceptances for different projection intervals.

Fig. 2 shows that the level of KS acceptances is greatly reduced with increasing length of the projection period, especially when using the NS algorithm. But the results of the CB method are also affected by the period length. As expected, the error index also increases with increasing length of the projection period.

The results indicate the superiority of the NS method for all the evaluated projection intervals when compared with the CB approach.

Fig. 3 shows a typical example of the observed and estimated cumulative distributions for both algorithms for the plot 82602, for projection periods of 1 and 5 years. The effect of the projection interval on the accuracy in the diameter distribution estimation can be observed.

Table 2
Acceptance percentage for the KS statistic ($\alpha = 0.01$) for the total number of projections ($n = 90$)

Method	Number of distributions accepted based on the KS criterion
NS	81 (90%)
CB	70 (78%)

Table 3
Mean values for the MAD (KS) and the index error for all the observations ($n = 90$)

Method	Statistic			
	MAD		Error index	
	Mean	S.D.[a]	Mean	S.D.
NS	0.034	0.0125	0.206	0.0838
CB	0.037	0.0165	0.211	0.0776

[a] Standard deviation.

4.2. Merchantable volume projection

The comparison between the observed and estimated volumes through the projection methods was based on measures of bias (mean residual) and error (mean absolute residual), which are presented in Table 4.

The two algorithms show relatively low values, both of error and bias. However, the NS method exhibits greater accuracy in the estimation for the total and merchantable volume.

Although the error and bias were low for both methods at stand level, some differences were detected

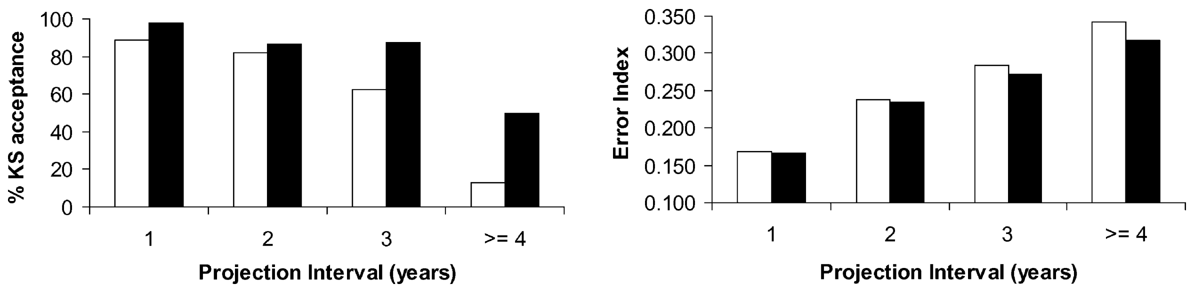

Fig. 2. Acceptance percentage of the KS test (left) and error index (right) for different projection intervals (white columns: CB method; black columns: NS method).

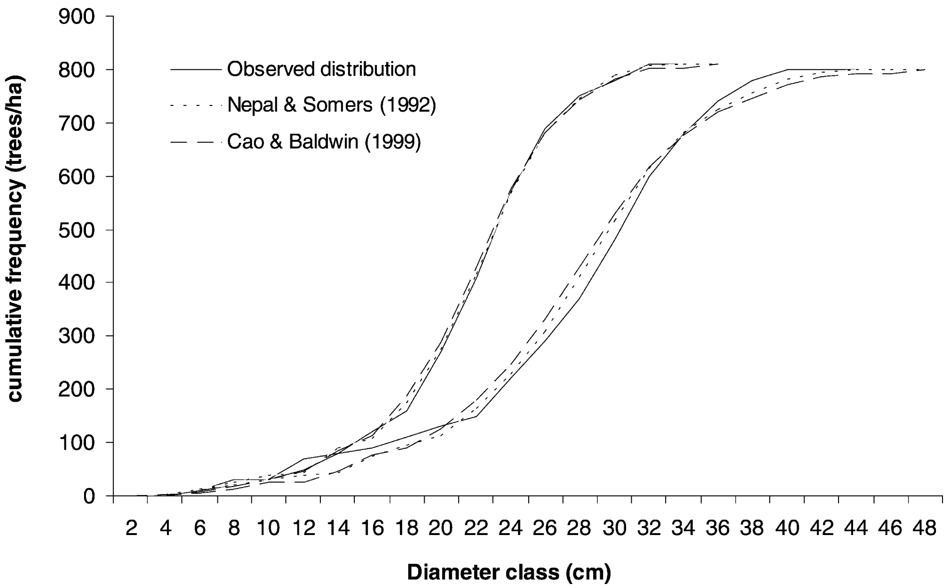

Fig. 3. Comparison of observed and estimated cumulative diameter distributions for both methods for projection intervals of 1 and 5 years (plot 82602).

Table 4
Mean values of error and bias for the evaluation of total volume and merchantable volume until a top diameter of 10 cm over all the observations ($n = 90$)

Method	Total volume (m^3/ha)		Merchantable volume[a] (m^3/ha)	
	Error	Bias	Error	Bias
NS	0.145	−0.020	0.714	0.238
CB	0.192	−0.141	1.379	0.601

[a] Volume until a top diameter of 10 cm.

when the volume distributions are analyzed at diameter classes level. This could be an important factor when estimating the volume of certain products which are defined by log size. Fig. 4 shows the estimated values obtained at stand level which demonstrate the high accuracy of both methods.

When inspecting the volume distribution by diameter classes, differences can be noted between the observed and estimated values for a projection period of 5 years. This points to a need for further investigations in the important area of timber product modeling.

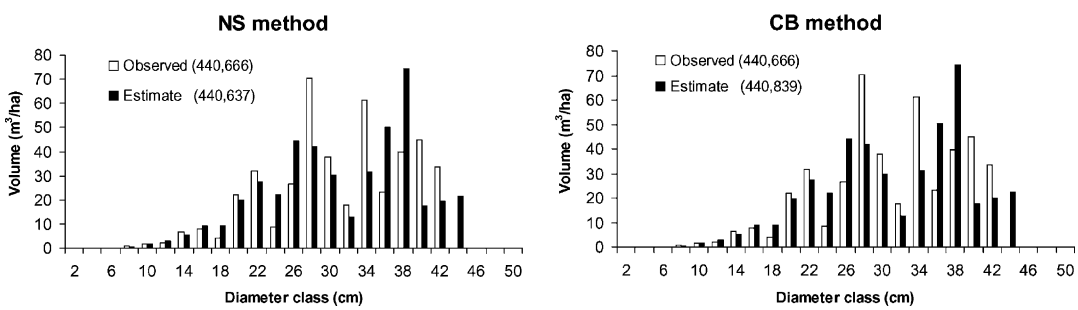

Fig. 4. Distribution of observed and estimated total volumes by diameter class for both projection methods, considering a projection interval of 5 years (plot 82602).

5. Conclusion

Based on the available evidence it can be concluded that both stand table projection methods are suitable for application in young *E. nitens* (Maiden) stands growing in central Chile. In all the analyzed projection periods, the method proposed by Nepal and Somers (1992) showed a slightly better performance when compared with the method of Cao and Baldwin (1999). The last-named approach was more sensitive to the length of the projection period, especially when the period extended over 4 years or more. Given short projection intervals, both methods showed a high accuracy in diameter distribution projections when projection periods did not exceed 3 years.

It appears that the NS method could also be usefully employed in irregular stands with multimodal diameter distributions, but further studies with other data sets would have to be done to evaluate this assumption.

Timber product modeling is essential for economic comparison of alternative silvicultural strategies. In this analysis, the projections of volume over diameter class distributions were not as satisfactory as the diameter distribution forecasts which points to a need for further investigations to improve the effectiveness of timber product modeling.

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